[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1130 **D**

Unique Paper Code : 2352571101

Name of the Paper : DSC: Topics in Calculus

Name of the Course : **B.A. / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **Two** parts from each question.
3. **All** questions carry equal marks.

1. (a) Give the $\in - \delta$ definition of the limit of a function.

   Use it to show that $\lim_{x \to 2} x^2 = 4$.

   (b) Define continuity of a function at a point x = c. Show that the function

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0 & x = 0 \end{cases}$$ is continuous at 0.

(c) Prove that the function $f(x) = \begin{cases} \frac{x}{1+e^{1/x}}, & x \neq 0 \\ 0 & x = 0 \end{cases}$ is continuous but not differentiable at $x = 0$.

2. (a) Show that the n^{th} differential coefficient of $\tan^{-1}x$ is $(-1)^{n-1}(n-1)!\sin^n\varnothing \sin n\varnothing$ where $\varnothing = \tan^{-1}x$.

(b) State Leibnitz theorem for finding n^{th} differential coefficient of product of two functions. Use it to prove that if $y = \tan^{-1}x$, then

$$(x^2+1)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0.$$

(c) State Euler's theorem and use it to prove that if

$$z = \log\frac{(x^4+y^4)}{(x+y)} \text{ then } x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 3.$$

3. (a) State Rolle's theorem. Give its geometrical interpretation. Verify it for

$f(x) = (x-1)^2(x+1)^3$ in $[-1,1]$.

(b) State Maclaurin's theorem. Also, find the Maclaurin's series for $f(x) = \cos x$.

(c) Find a, b, c so that $\lim_{x \to 0} \frac{ae^{x} - b\cos x + ce^{-x}}{x \sin x} = 2$.

4. (a) State Lagrange's mean value theorem and use it to show that

$$1 + x < e^{x} < 1 + xe^{x}, \quad x > 0.$$

(b) Prove $\sin ax = ax - \frac{a^3x^3}{3!} + \cdots + \frac{a^{n-1}x^{n-1}}{(n-1)!}$

$$\sin\left(\frac{(n-1)\pi}{2}\right) + \frac{a^n x^n}{n!} \sin\left(a\theta x + \frac{n\pi}{2}\right)$$

(c) Find the following limits

(i) $\lim_{x \to 0} \frac{\log x}{\cot x}$ (ii) $\lim_{x \to \pi/2} (\sec x - \tan x)$

5. (a) Find the double points of the curve

$$y(y - 6) = x^2(x - 2)^3 - 9$$

and analyse their nature.

(b) Trace the curve $y^2(2 + x) = x^2(2 - x)$.

(c) Find the reduction formula for $\int \sin^n x \, dx$ and hence prove

$$\int \sin^4 x \, dx = \frac{-1}{4}\sin^3 x \cos x - \frac{3}{8}\sin x \cos x + \frac{3x}{8} + c.$$

6. (a) When is a curve said to be concave up and concave down at a point P. Find the range of values of x for which $y = 3x^5 - 40x^3 + 3x - 20$ is concave up or down.

(b) Trace the curve $r^2 = 4\cos 2\theta$.,

(c) Prove $\int \sec^n x \, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x \, dx$