[This question paper contains 8 printed pages.]

Your Roll No..............

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Sr. No. of Question Paper : 2301

Unique Paper Code : 62354443

Name of the Paper : Analysis (LOCF)

Name of the Course : **B.Sc. (Prog.)**

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory.

3. Attempt any **two** parts from each question.

4. **All** questions carry equal marks.

1. (a) State and prove Archimedean property of real numbers.

(b) Let A and B be two nonempty bounded sets of positive real numbers, and let

$$C = \{xy : x \in A \text{ and } y \in B\}.$$

Show that C is bounded and :

(i) $\text{Sup } C = \text{Sup } A \text{ Sup } B$

(ii) $\text{Inf } C = \text{Inf } A \text{ Inf } B$

(c) Let f be a function on $\mathbb{R}$ defined by

$$f(x) = \begin{cases} 1, & \text{when } x \text{ is rational} \\ -1, & \text{when } x \text{ is irrational} \end{cases}$$

Show that f is discontinuous at every point of $\mathbb{R}$.

(d) Define Uniform continuity of a function f on an interval I. Show that the function f defined by $f(x) = \sqrt{x}$ is uniformly continuous in the interval [1,3].

2. (a) Show that the sequence $<r^n>$ converges to zero if $|r| < 1$. Discuss other cases also.

(b) Show that $\lim_{n\to\infty} \sqrt[n]{n} = 1$.

(c) Define the limit of a sequence of real number. Let $<a_n>$ be a sequence of positive terms such that $<a_n> \to a$. Then prove that $\sqrt{a_n} \to \sqrt{a}$.

(d) If f and g be two real functions defined on some neighbourhood of c such that $\lim_{x\to c} f(x) = l$,

$\lim_{x \to c} g(x) = m$, then show that

$$\lim_{x \to c} (fg)x = \lim_{x \to c} f(x) \lim_{x \to c} g(x) = lm.$$

3. (a) Let $<a_n>$ be a sequence of real numbers such that $a_n \neq 0$ for all n and

$$\lim_{n \to \infty} \left(\frac{a_{n+1}}{a_n} \right) = 1, \text{ where } |L| < 1.$$

Show that $\lim_{n \to \infty} a_n = 0$. Also, Deduce that

$$\lim_{n \to \infty} 2^{-n} n^2 = 0.$$

(b) Define Cauchy sequence. Use Cauchy's General Principle of Convergence to show that the sequence $<a_n>$ defined by

$$a_n = 1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1}$$

does not converge.

(c) Prove that the sequence $<S_n>$ defined by the recursion formula : $S_{n+1} = \sqrt{3S_n}$, $S_1 = 1$, $n \geq 2$ converges to 3.

(d) Give an example of a bounded subset $S \neq \phi$ of $\mathbb{R}$ whose least upper bound and greatest lower bound belong to S^c.

4. (a) State Cauchy's n^{th} root test for the series. Use this test to check the convergence of the series

$$\frac{n^{n^2}}{(n+1)^{n^2}}$$

(b) State and prove the necessary condition for the convergence of a series. Is the converse holds true? Justify your answer.

(c) Test the convergence of the following series

(i) $\sum_{n=1}^{\infty} \frac{r^n}{n!}$ where r is any positive number.

(ii) $\sum_{n=1}^{\infty} \frac{n^n x^n}{n!} \, (x > 0)$

(d) State the D' Alembert ratio test and Raabe's test for the convergence of the series. Test the convergence of the series

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} x^n$$

5. (a) Define continuity of a real valued function at a point.

Show that the function defined as $\begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 6, & x = 3 \end{cases}$

is continuous at x=3.

(b) Show that every absolutely convergent series is convergent. Is the converse holds true? Give example.

(c) Show that the function f defined on [a, b] as

$$f(x)=\begin{cases}0 & \text{when x is rational}\\ 1 & \text{when x is irrational}\end{cases}$$

is not Riemann integrable.

(d) Show that the series $1+r+r^2+r^3+\cdots$ $(r>0)$ converges if $r<1$ and diverges if $r>1$.

6. (a) Define Riemann integrability of a bounded function f on a bounded closed interval [a,b],

(b) Use the definition of Riemann integrability to prove

$$\int_0^b x^2\,dx=\frac{b^3}{3}$$

(c) State Non-uniform Continuity Criteria of a function defined on $A \subseteq R$. Use it to prove $f(x) := \frac{1}{x}$ is not uniform continuous on $A = \{x \in R \mid x > 0\}$.

(d) Define supremum and infimum of the set $S \subseteq \mathbb{R}$.

Find the supremum and infimum of the set

$$S = \{1 - (-1)^n/n : n \in \mathbb{N})$$