[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 2363 **H**

Unique Paper Code : 62354443

Name of the Paper : Analysis

Name of the Course : B.A. (Programme) – Core Course

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. Part of questions to be attempted together.
4. **All** questions carry equal marks.
5. Use of calculator not allowed.

1. (a) Define the upper bound and lower bound of a non-empty subset of $\mathbb{R}$. Find the upper bound and lower bound for the following sets, if they exist. (6.5)

   (i) $\{-1, 1, -2, -2, \ldots, -n, n \ldots\}$

(ii) $\left\{0, \frac{1}{3}, \frac{2}{4}, \ldots, \frac{n-1}{n+1}, \ldots\right\}$

(iii) $\left\{2, \frac{3}{2}, \frac{4}{3}, \ldots, \frac{n+1}{n}, \ldots\right\}$

(iv) $\{1^2, 2^2, 3^2, \ldots, n^2, \ldots\}$

(b) Define continuity of a real valued function at a point. Show that the function defined as

$$f(x)=\begin{cases}\frac{x^2-9}{x-3}, & x \neq 3 \\ 6, & x=3\end{cases}$$

is continuous at $x = 3$. (6.5)

(c) State and Prove the Archimedean property of real numbers. (6.5)

2. (a) Show that the function $f(x) = x^2$ is uniformly continuous in the interval $[-2, 2]$. (6)

(b) Define the Sequential Criterion of limit. Show that $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ does not exist in $\mathbb{R}$. (6)

(c) State the Sequential Criterion of continuity. Define $f: \mathbb{R} \to \mathbb{R}$ by

$$f(x)=\begin{cases}1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational}\end{cases}$$

Show that f is not continuous at any point of $\mathbb{R}$. (6)

3. (a) Use the definition of limit of a sequence to establish the following limits :

(i) $\lim_{n\to\infty}\left(\frac{n}{n^2+1}\right)=0$. (3.5)

(ii) $\lim_{n\to\infty}\left(\frac{3n+1}{2n+5}\right)=\frac{3}{2}$. (3)

(b) Discuss the convergence of the sequence $\left(\frac{11^{2n}}{7^{3n}}\right)$. (6.5)

(c) Show that the sequence (x_n) defined by

$$x_1=a>0,\ x_{n+1}=\frac{2x_n}{1+x_n},\quad n>1,$$

is bounded and monotone. Also, find the limit. (6.5)

4. (a) Give an example of an unbounded sequence that has a convergent subsequence. (6)

(b) Show that the sequence (x_n) defined by

$$x_n=1+\frac{1}{6}+\frac{1}{11}+\cdots+\frac{1}{5n-4}$$

is not Cauchy. (6)

(c) Calculate the value of $\sum_{n=2}^{\infty}\left(\frac{2}{7}\right)^n$. (6)

5. (a) State D'Alembert's ratio test for an infinite series. Test for convergence of the series

$$\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}.$$ (6.5)

(b) Test for convergence the following series :

(i) $\sum_{n=1}^{\infty} \left(1+\frac{1}{\sqrt{n}}\right)^{-n^{\frac{3}{2}}}$ (3.5)

(ii) $\frac{1^2.2^2}{1!}+\frac{2^2.3^2}{2!}+\frac{3^2.4^2}{3!}+\cdots$ (3)

(c) Show that the greatest integer function f(x) = [x] is Riemann integrable on [0, 4] and $\int_0^4 [x]dx = 6$. (6.5)

6. (a) Test the convergence and absolute convergence of the series :

(i) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n\sqrt{n}}$ (3)

(ii) $\frac{1}{1.2}-\frac{1}{3.4}+\frac{1}{5.6}-\frac{1}{7.8}+\cdots$. (3)

(b) State Leibnitz's test for an alternating series. Test for convergence the series :

$1-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{7}}+\cdots$ (6)

(c) Show that every monotonic function on [a, b] is Riemann integrable on [a, b]. (6)