[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 986 **G**

Unique Paper Code : 2352201102

Name of the Paper : DSC: Elements of Discrete Mathematics

Name of the Course : **B.A. (Prog.)**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **two** parts from each question.

3. **All** questions are compulsory.

4. Marks are indicated.

1. (a) Determine the following :

(i) Compute the truth table of the statement

$(p \Rightarrow q) \equiv (\sim q \Rightarrow \sim p)$.

(ii) If $p \Rightarrow q$ is false, then determine the truth value of $(\sim p) \vee (p \Leftrightarrow q)$. Explain your answer. (7.5)

(b) Let $A = \mathbb{Z}$ (the set of integers). Define the following relation R on A :

$a \, R \, b$ if and only if $|a - b| = 2$.

Determine whether the relation R on A is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive. Is R an equivalence relation on A? (7.5)

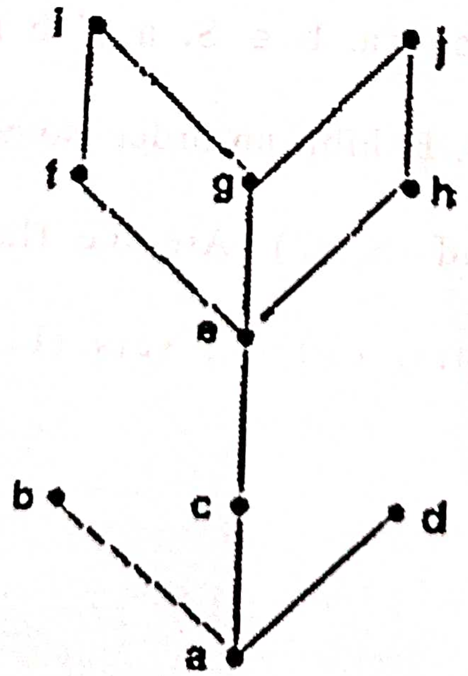
(c) Prove by mathematical induction that if $A_1, A_2, \ldots, A_n$ are any n sets, then

$$\overline{\bigcap_{i=1}^{n} A_i} = \bigcup_{i=1}^{n} \bar{A}_i$$ where $\bar{A}_i$ denote the complement of the set A_i. (7.5)

2. (a) Let X = {1, 2, 3}. Consider the partial ordered set (L, ≤) where L = P(X) is the power set of X and ≤ is defined as, U ≤ V if and only if U ≤ V ∀ U, V ∈ L. Also consider partial ordered set S of all positive divisors of 30, with respect to the order that for any a, b ∈ S, a ≤′ b if and only if a divides b. Exhibit an order isomorphism between (L, ≤) and (S, ≤′). Are the Hasse diagrams of two partial ordered sets (L, ≤) and (S, ≤′) identical? (7.5)

(b) Let N_0 be the set of whole numbers equipped with the partial order $\leq$ of divisibility defined as $a \leq b$ if and only if a divides b. Draw a Hasse diagram for the subset $P = \{2,3,12,18\}$ of $(N_0, \leq)$. How many maximal and minimal elements are there in $(P, \leq)$? (7.5)

(c) Find the lower and upper bounds along with greatest lower and least upper bound of the subsets {c, e}, {b, i) in the following Hasse diagram. (7.5)



3. (a) Determine whether the relation $(\mathbb{Z}, \leq)$ on the set of all integers with the order "less than equal to" is a lattice. (7.5)

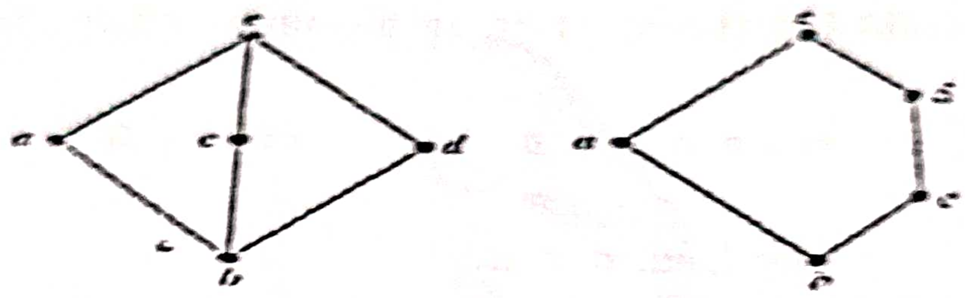
(b) Let $(L, \wedge, \vee)$ be an algebraic lattice. Show that $m \leq n \Rightarrow l \wedge m \leq l \wedge n$ and $l \vee m \leq l \vee n$, for any $l, m, n \in L$. (7.5)

(c) Define a sublattice of a lattice L. Show that the interval $[x, y] = \{l \in L : x \leq l \leq y\}$, is a sublattice for any two elements $x, y \in L$ with $x \leq y$.

(7.5)

4. (a) Define a distributive lattice. Prove that a homeomorphic image of a distributive lattice is distributive. (7.5)

(b) Does the following diamond and pentagonal lattices satisfy the distributive laws? (7.5)



(c) Define a complemented lattice. Also show that $(P(M), \cap, \cup)$ is a complemented lattice for the power set P(M) of a non-empty set M.

(7.5)

5. (a) What is Karnaugh map? Use Karnaugh map diagram to find a minimal form of the function $f(x,y,z,t) = x\overline{y} + xyz + \overline{x}\,\overline{y}\,\overline{z} + \overline{x}yz\overline{t}$. (7.5)

(b) Find the DN form and CN form of the following Boolean functions

$$f(x,y,z)=x\overline{y}+x\left(\overline{yz}\right)+xyz$$ (7.5)

6. (a) Let $f(x,y,z)=xy\overline{z}+\overline{x}yz+\overline{x}y\overline{z}$. Find the implicants, prime implicants and essential prime implicants of f(x, y, z).

$$\overline{\left(x\left(\overline{y}\,\overline{z}\right)\right)}=\overline{x}+(y+z)(\overline{y}+\overline{z}).$$ (7.5)

(b) Construct a logic circuit corresponding to Boolean function

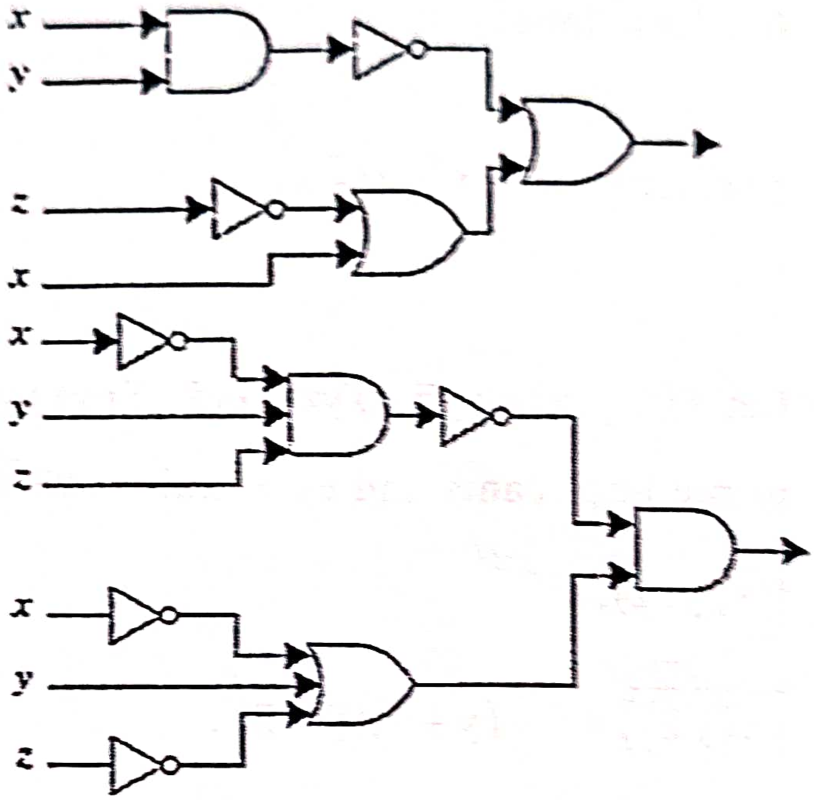
(i) $f(x, y, z) = xyz' + yz' + x'y$

(ii) $f(x, y, z, w) = (x + y)(x' + z) + (z + w)'$

(7.5)

(c) Determine the output of each of these circuits

(7.5)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