[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2253 G

Unique Paper Code : 2354002001

Name of the Paper : DIFFERENTIAL EQUATIONS

Name of the Course : COMMON PROG GROUPS (Generic Elective)

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** question by selecting **two** parts from each question.

3. **All** questions carry equal marks.

4. Use of Calculator not allowed.

1. (a) Determine the values of p for which the function g defined by $g(x) = x^p$ is a solution of the differential equation

$$x^3\frac{d^3y}{dx^3}+2x\frac{d^2y}{dx^2}-10x\frac{dy}{dx}-8y=0.$$ (7.5)

(b) Solve the equation

$(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0.$ (7.5)

(c) Solve the differential equation

$2r(s^2 + 1)dr + (r^4 + 1)ds = 0.$ (7.5)

2. (a) Solve the differential equation

$$\frac{dx}{dt}+\frac{x}{t^2}=\frac{1}{t^2}.$$ (7.5)

(b) Solve the differential equation

$(5xy + 4y^2 + 1)dx + (x^2 + 2xy)dy = 0,$

by first finding an integrating factor. (7.5)

(c) Determine the value of K such that the parabolas $y = c_1x^2 + K$ are the orthogonal trajectories of the family of ellipses $x^2 + 2y^2 - y = c_2$. (7.5)

3. (a) Find the solution of the differential equation

$$4\frac{d^2y}{dx^2}+4\frac{dy}{dx}+37y=0,\ \ y(0)=2,\ y'(0)=-4.$$ (7.5)

(b) Find the general solution of the differential equation

$$y'' - 5y' + 6y = 4e^{2x},$$

using method of undetermined coefficients.

(7.5)

(c) Use the method of variation of parameters to find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + y = \sec^2 x. \quad (7.5)$$

4. (a) Find the general solution of the differential equation

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3. \quad (7.5)$$

(b) Solve the linear system

$$\frac{dx}{dt} + \frac{dy}{dt} + 2y = \sin t, \quad \frac{dx}{dt} + \frac{dy}{dt} - x - y = 0 \quad (7.5)$$

(c) Show that $x = 2e^{2t}$, $y = -3e^{2t}$ and $x = e^{7t}$, $y = e^{7t}$ are two linearly independent solutions on every interval $a \le t \le b$ of the homogeneous linear system

$$\frac{dx}{dt} = 5x + 2y, \quad \frac{dy}{dt} = 3x + 4y$$

Write the general solution. (7.5)

5. (a) Apply $\sqrt{u} = v$ and $v(x, y) = f(x) + g(y)$ to solve

$$x^4 u_x^2 + y^2 u_y^2 = 4u. \quad (7.5)$$

(b) Find the solution of the initial-value systems

$$u_t + u u_x = e^{-x} v, \; v_t - a v_x = 0,$$

with $u(x, 0) = x$ and $v(x, 0) = e^x$. (7.5)

(c) Determine the general solution of

$$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2. \quad (7.5)$$

6. (a) Given that the parabolic equation

$$u_{xx} = au_t + bu_x + cu + f,$$

where the coefficients are constants, by the substitution $u = v\, e^{\frac{1}{2}bx}$ and for the case $c = -(b^2/4)$, show that the given equation is reduced to the heat equation

$$v_{xx} = a v_t + g,$$

where $g = f e^{-bx/2}$. (7.5)

(b) Find the solution of the Cauchy problem

$$x u_x - y u_y + y^2 u = y^2. \quad (7.5)$$

(c) Apply $v = \ln u$ and then $v(x, y) = f(x) + g(y)$ to solve

$$x^2 u_x^2 + y^2 u_y^2 = (x y u)^2. \quad (7.5)$$