

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2032

**F**

Unique Paper Code : 2354001201

Name of the Paper : Analytic Geometry

Name of the Course : **COMMON PROG GROUP**

Semester / Type : II / GE

Duration : 3 Hours

Maximum Marks : 90

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each Question.
3. Part of the questions to be attempted together.
4. Each question carries equal marks.
5. Use of Calculator not allowed.

**Note:- All field as mentioned in the Performa are essential.**

P.T.O.

2032

1. (a) The parabola  $y^2 = 8x$  is shifted down 2 units and right 1 unit to generate the new parabola;

(i) Find the equation of new parabola.

(ii) Find new parabola's vertex, focus and directrix.

(iii) Plot the new vertex, focus and directrix and sketch in the parabola.

(b) (i) Decide whether the equation

$$9x^2 + 6xy + y^2 - 12x - 4y + 4 = 0$$

represents an ellipse, a parabola, or a hyperbola.

(ii) Write the basic steps, describe and draw the graph of equation

$$(x + 2)^2 = -(y + 2).$$

- (c) The coordinate axes are to be rotated through an angle  $\alpha$  to produce an equation for the curve  $x^2 + xy + y^2 = 1$  that has no cross product term. Find  $\alpha$  and the new equation. Identify the curve and plot the same.
2. (a) Sketch the graphs of the hyperbola  $y^2 - x^2 = 1$  showing their vertices, foci and asymptotes.
- (b) Find the equation of the hyperbola with vertices  $(0, \pm 8)$  and asymptotes  $y = \pm \frac{4}{3}x$ .
- (c) Find an equation of the parabola traced by a point that moves so that its distance from  $(2, 4)$  is the same as its distance to x-axis.

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3. (a) (i) Find the orthogonal projection of  $\vec{v} = i + j + k$  on  $\vec{b} = 2i + 2j$ , and then find the vector component of  $\vec{v}$  orthogonal to  $\vec{b}$ .

(ii) Find two-unit vectors that are orthogonal to both

$$\vec{u} = 3i + 2j - k \text{ and } \vec{v} = -i + 3j + k.$$

(b) (i) Use a scalar triple product to find the volume of the parallelepiped that has

$$\vec{u} = (2, -6, 2), \vec{v} = (0, 4, -2) \text{ and } \vec{w} = (2, 2, -4)$$

as adjacent edges.

(ii) Find the acute angle of intersection between the two planes

$$x + 2y - 2z = 5 \text{ and } 6x - 3y + 2z = 8.$$

(c) Find the magnitude and the equations of the line of shortest distance between the two lines

$$\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5} \quad \text{and} \quad \frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}.$$

4. (a) (i) Find parametric equations of the line L passing through the points P(2, 4, -1) and Q(5, 0, 7). Where does the line intersect the xy-plane?

(ii) Find the distance D between the point (0, 1, 5) and the plane  $3x + 6y - 2z - 5 = 0$ .

(b) Find the equation of the plane passing through the points P(-2, 1, 4), Q(1, 0, 3) and that is perpendicular to the plane  $4x - y + 3z = 2$ .

(c) Find the direction cosines of the line which is perpendicular to the lines with direction cosines proportional to  $(1, -1, 2)$  and  $(2, 1, -1)$ .

5. (a) Through a point P three mutually perpendicular straight lines are drawn; one passes through a fixed-point C on the z-axis, while the others intersect the x-axis and y-axis, respectively; show that the locus of P is a sphere of which C is the centre.

(b) (i) Derive the equation of the tangent plane at any point  $(\alpha, \beta, \gamma)$  to the sphere

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0.$$

(ii) Show that the plane  $lx + my + nz = p$  will touch the sphere

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0,$$

$$\text{if } (ul + vm + wn + p)^2 = (l^2 + m^2 + n^2)$$

$$(u^2 + v^2 + w^2 - d).$$

(c) Find the centre and radius of the circle :

$$x + 2y + 2z = 15, \quad x^2 + y^2 + z^2 - 2y - 4z = 11.$$

6. (a) Find the equation of the cone whose generators pass through the point  $(\alpha, \beta, \gamma)$  and have their direction cosines satisfying the relation  $al^2 + bm^2 + cn^2 = 0$ .

(b) Show that the locus of the line of intersection of perpendicular tangent planes to the cone  $ax^2 + by^2 + cz^2 = 0$ , is the cone

$$a(b + c)x^2 + b(c + a)y^2 + c(a + b)z^2 = 0.$$

- (c) Find the equation of a right circular cylinder of radius 2 whose axis is the line

$$\frac{x-1}{2} = (y-2) = \frac{z-3}{2}.$$