Unique Paper Code   : 32347616

Name of the Paper   : Advanced Algorithms

Name of the Course   : B.Sc. (Hons:) Computer Science

Year of Admission   : (For Admissions of 2019 onwards)

Semester   : VI

Duration : 3 Hours   Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Question No. 1 is compulsory in **Section-A**.

3. Attempt any **four** questions from **Section-B**.

4. Parts of a question should be attempted together.

Section A

1. (a) Consider that all the elements of a singly linked list are inserted in the list in ascending order. What

is the average case time complexity to search an element in this singly linked list. Given O(n) extra space can we reduce average case search time to O(log n)? If yes, Discuss your algorithm. If No, Justify your answer.

(5)

(b) In a text corpus, frequencies of the characters are as shown below.

| Characters	A	B	C	D	E	F	G
Frequencies	12	14	12	3	7	22	2

If Huffman Coding is used for data compression of the above corpus, determine :

(i) Huffman Code for each character

(ii) Average code length

(iii) Length of Huffman encoded message (in bits)

(5)

(c) Consider there are m job applicants and n jobs. Each applicant has a subset of jobs that he/she is interested in. Each job opening can only accept one applicant and a job applicant can be appointed for only one job. Find an assignment of jobs to applicants such that as many applicants as possible get jobs. Write an O(m*n) algorithm to solve this problem.

(5)

(d) Prove that 3-SAT problem is polynomial time reducible to the Clique problem. (5)

(e) Consider a network of pipes with a flow of liquid inside the pipes. Each pipe has a certain capacity of liquid it can transfer at an instance. Calculate how much liquid can be flowed from the source S to the sink T at an instance using the network below. Illustrate all the steps showing the residual graphs. (5)

(f)

**Total Cases**

(Linear Scale)

P.T.O.
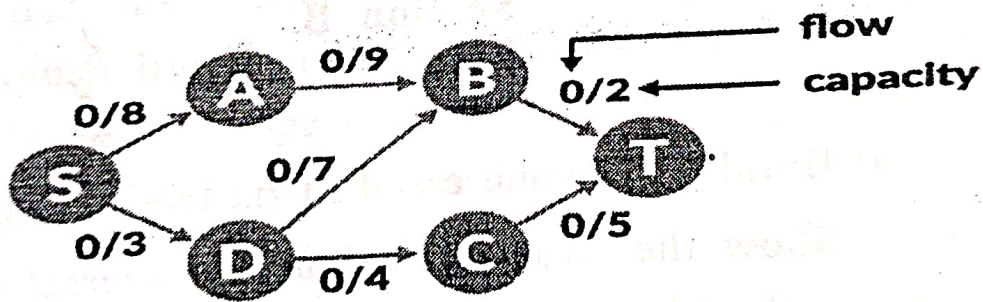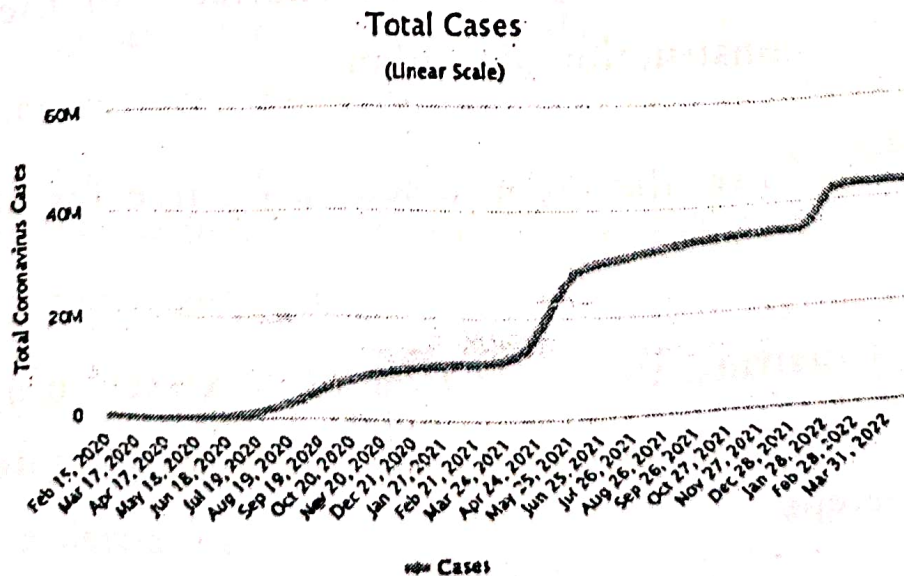
Consider the above Graph, which contains month-wise COVID-19 Total Cases in India from Feb 2020 to 31 March 2022. Discuss which algorithm is better among linear regression or segmented least squares, in order to predict new total cases in month May 2022. Justify your answer.   (5)

(g) Prove that A red-black tree with n nodes has height $h \leq 2 \lg(n + 1)$.   (5)

Section B

2. (a) Build an uncompressed suffix trie for "banana$". Show the structure and node traversal path for each suffix insertion. Mark the suffix links that are actually used as shortcuts in the efficient construction algorithm.   (6)

(b) Draw the compressed suffix tree for "banana$".   (4)

3. Illustrate the operation of "Extract min" on the following fibonacci heap. Show all the intermediate steps.   (10)
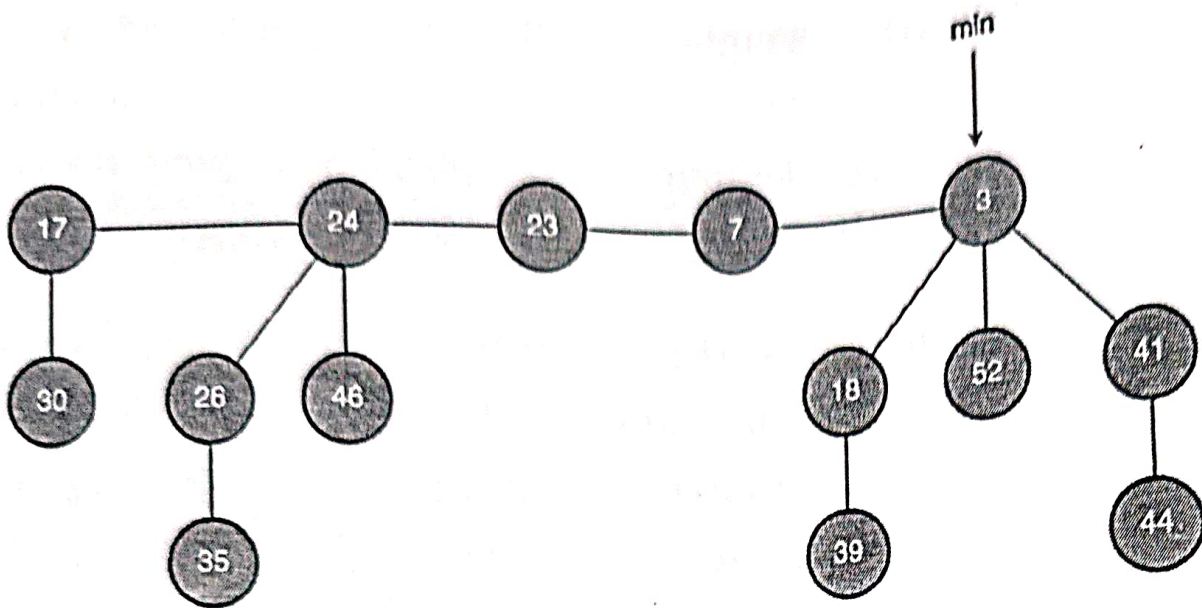
4. (a) There are 4 friends Rohan, Shyam, Manish and Ayush living at Gurugram, Dwarka, Noida and Ghaziabad respectively. They have to appear in CUET-2022 Exam which is scheduled at 4 different locations Rohini, Lakshminagar, South Extension and Karol Bagh respectively. Rohan have taken admit card of all 4 students. So they have decided to meet at a centre point i.e. Rajiv Chowk. After meeting they need to go to their respective centres. Draw a graph to represent this problem. Also Show that how we can solve this problem by Dijkstra Algorithm.

(5)

(b) Given a directed weighted graph. You are also given the shortest path from a source vertex 's' to a destination vertex 't'.

    (i) If weight of every edge is increased by 15 units, does the shortest path remain same in the modified graph? Justify your answer by giving an example.

    (ii) Does the shortest path change when weights of all edges are multiplied by 5? Justify your answer by giving an example. (5)

5. (a) Write Karatsuba's algorithm of fast integer multiplication for multiplying two large integers in $O(n^{1.59})$ time. (5)

    (b) Consider two numbers A = 11111001 and B = 10111111. Illustrate the working of Karatsuba's algorithm for A X B. (5)

6. (a) Sudoku is a puzzle game which is made up of a board of 9X9 grid, divided into nine 3X3 subgrids. For Sudoku, a solution is valid if the following constraints are true.

    (i) Each row has unique numbers from 1-9.

    (ii) Each column has unique numbers from 1-9.

(iii) Each sub-grid of 1-9 has the numbers 1-9.

Initially, some of the grids are partially filled in. Here is an example of a starting board.

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Give a backtracking solution of the above problem. (5)

(b) Is it possible to find longest path between two vertices in a graph in polynomial time? Justify your answer. Is it possible to find smallest path between two vertices in a graph in polynomial time? Justify your answer. (5)

7. (a) Consider the problem of interval scheduling, if S_i is starting time of the activity and f_i is finishing time of the activity. In order to solve interval scheduling problem consider the following three greedy strategies.

(i) Sort activities in ascending order according to s_i and select the activity with smallest s_i first.

(ii) Sort activities in ascending order according to f_i and select the activity with smallest f_i first.

(iii) Sort activities in ascending order according to $f_i - s_i$ and select the activity with smallest $f_i - s_i$ first.

Justify which strategy/strategies always gives optimal result. Also justify the reason for not obtaining the optimal results by other strategy/ strategies using an example. (5)

(b)

Team	Wins	Losses	Games Left	Remaining Games Against			
				GT	RCB	RR	LSG
Gujrat Titans (GT)	83	71	8	-	1	6	1
Royal Challengers Bangalore (RCB)	80	79	3	1	-	0	2
Rajasthan Royals(RR)	78	78	6	6	0	-	0
Lucknow Super Giants(LSG)	77	82	3	1	2	0	-

The IPL is a professional club cricket league played by different teams in India. Consider the current status (given in the above table) of the teams. The above table shows the 4 teams information: Gujrat Titans (GT), Royal Challengers Bangalore (RCB), Rajasthan Royals (RR), Lucknow Super Giants (LSG). It shows the win/ loss record for each team, the number of games each team has left, and who these remaining games are to be played against.

(i) Convert this table into flow network.

(ii) Show that RCB will never top the table.

(5)

8. (a) Given a 3×3 board with 8 tiles (every tile has one number from 1 to 8) and one empty space. The objective is to place the numbers on tiles in order using the empty space. We can slide four adjacent (left, right, above and below) tiles into the empty space. It is not possible to solve an instance of 8 puzzle if number of inversions is odd in the input state. Which of the following tiles states are solvable? Justify your answer. Also Compute all inversions in both tiles. (5)

(i)

1	8	2
	4	3
7	6	5

(ii)

8	1	2
	4	3
7	6	5

(b) Define the following terms with 1 example of each problem?

    (i) Class P

    (ii) Class NP

    (iii) Class NP-Hard

    (iv) Class NP-Complete

    (v) Class co-NP (5)